

# 一种基于DOS攻击观测器的区间二型T-S模糊系统的安全控制方法

何家伟<sup>1</sup>, 张孙杰<sup>2</sup>

<sup>1</sup>上海理工大学理学院, 上海

<sup>2</sup>上海理工大学光电信息与计算机工程学院, 上海

收稿日期: 2022年5月29日; 录用日期: 2022年6月21日; 发布日期: 2022年6月30日

## 摘要

本文重点讨论了在DOS攻击下, 具有分布式延迟的区间类型-2 (IT2)T-S模糊离散时间系统的抗异常值观测器和控制器。DOS攻击发生在传感器通道和观测器通道之间。为了解释随机发生的DOS攻击对控制结构的影响, 构建了一个安全性能的新指导。参数不确定性和DOS攻击的随机事件通过两组具有概率分布的伯努利随机变量进行控制。构建了一个抗异常值的基于观测器的安全控制(OBSC), 以诱导创新的饱和结构, 从而减轻对DOS进攻的估计方面的消极影响。该方案的目标是设计一个OBSC, 在实现理想的安全水平的同时, 能够保证估计误差的有界性。最终, 提出了一个数值模拟样本来证明上述控制项目的实用性和有效性。

## 关键词

分布时滞, 区间二型模糊系统, DOS攻击

# An Internal Type-2 T-S Fuzzy Based on DOS Attacks Observer System with a Security Control Method

Jiawei He<sup>1</sup>, Sunjie Zhang<sup>2</sup>

<sup>1</sup>College of Science, University of Shanghai for Science and Technology, Shanghai

<sup>2</sup>School of Optical Information and Computer Engineering, University of Shanghai for Science and Technology, Shanghai

Received: May 29<sup>th</sup>, 2022; accepted: Jun. 21<sup>st</sup>, 2022; published: Jun. 30<sup>th</sup>, 2022

## Abstract

This article focuses on the outlier-resistant observer-based controller for the interval type-2 (IT2) T-S fuzzy discrete-time systems with distributed delays under DOS attacks. The DOS attack takes place between sensor channel and observer channel. A neoteric guidance for safety performance is constructed to explain the randomly happening DOS offensive on the control structure. The random events of parameter uncertainty and DOS offensives are controlled via two sets of Bernoulli stochastic variables with foregone probability distributions. One outlier-resistant observer-based security control (OBSC) is constructed to induce an innovation saturation structure so as to alleviate the passive influence on the DOS offensives about the estimation aspect. The goal of the solution is to design an OBSC that can ensure boundedness in estimation error while achieving the ideal level of safety. Ultimately, a quantitative value simulation sample is put forward to justify the practicality and validity of the above-mentioned control project.

## Keywords

Distributed Delay, Interval Type-2 Fuzzy Systems, DOS Attacks

Copyright © 2022 by author(s) and Hans Publishers Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

## 1. 引言

高木和杉野在 20 世纪 80 年代提出了 T-S 模糊模型(FM) [1] [2]。最初, T-S FM 模型只用于非线性函数的逼近方法, 所以它不是一个动态系统。后来, Cao 等人将原来的 T-S 调频模型拓宽为实际应用的动态模型, 并将其用于近似非线性动态系统。此后, 基于 T-S 模糊模型的稳定性研究和控制器方案得到了广泛的关注和发展。T-S 模糊模型是一个由平滑连接的线性子模型与非线性模糊权重组成的整体模型。在凸集下, T-S FM 能够以任何精度接近一些平滑的非线性函数。

传统的 T-S 调频通常假定模糊权重不包含不确定信息, 并采用平行分布补偿器(PDC)策略来设计反馈控制器[3] [4] [5]。最后, 利用 Lyaounov 稳定性理论得出控制器的存在条件。尽管 PDC 策略可以处理具有不确定成员函数(MFs)的 T-S 调频, 但它不能有效地利用隐藏在 MFs 中的不确定信息。在实际的工业应用中, 非线性系统往往伴随着不确定性, 例如, 不确定的参数, 包含不可测量的变量, 或者未知的干扰。因此, 当采用传统的 T-S 调频来逼近具有不确定参数的非线性结构时, 其模糊权重可能包含不确定参数, 不能直接采用 PDC 策略来安排模糊反馈控制器[6]。Type-1 T-S FM 能够在凸集中接近任何平滑的非线性函数, 并具有良好的局部线性化特性, 成为非线性系统稳定性分析和控制器设计的有力工具[7] [8]。然而, Type-1 T-S FM 不能高度处理不确定系统。2 型模糊集可以提高处理不确定性的能力, 将 2 型模糊逻辑扩展到 T-S 模型框架中, 具有重要的理论意义和应用价值。作为普通 2 型模糊集的特殊版本, IT2 模糊集计算简便, 且保留了 2 型模糊集处理不确定性的良好特性, 因此在实际应用中更有价值。与第一类 T-S 模糊系统不同, IT2 T-S 调频已被证明更适用于具有不确定参数的非线性系统。

显然, 由于受到 DOS 攻击的影响, 测量的输出可能会遇到突然的巨大扰动, 这反过来又可能导致所谓的测量异常值和估计属性的进一步恶化[9] [10]。例如, 没有经过充分处理的测量异常值很可能导致估算器中的创新值发生巨大变化, 从而损害估算的真实性。因此, 构建一个无视测量异常值的估算器是非

常必要的。这种估计器被称为抗异常值估计器。近年来, 抗异常值状态估计问题开始得到初步的研究热点, 并从文字上得到了一些好的结果。

## 2. 问题描述

### 2.1. 区间二型 T-S 模糊模型

考虑如下具有分布时滞和扰动的区间二型 T-S 模糊时变系统

系统规则  $i$ : 如果  $f_1(x(k))$  是  $\tilde{M}_1^i$ , 且  $f_2(x(k))$  是  $\tilde{M}_2^i$ , 且  $\dots$ , 且  $f_\psi(x(k))$  是  $\tilde{M}_\psi^i$ , 则

$$\begin{cases} x(k+1) = (A_i + \zeta(k)B(k))x(k) + F_i u(k) + D_i v(k) + H_i \sum_{h=1}^{\tilde{h}} \mathcal{G}_h x(k-h) \\ y(k) = C_i x(k) \\ z(k) = E_i x(k) \\ x(j) = \phi(j), j = \wp = \{-\tilde{h}, \dots, -1, 0\} \end{cases} \quad (1.1)$$

$x(k) \in R^{n_x}$ 、 $z(k) \in R^{n_z}$ 、 $y(k) \in R^{n_y}$ 、 $u(k) \in R^{n_u}$  分别表示系统的状态、测量输出、估计输出、控制输入;  $\tilde{h}$  是已知标量,  $\phi(j)$  是初始值,  $\mathcal{G}_h$  是正标量,  $A_i, C_i, D_i, E_i$  和  $H_i$  是适合维数的实矩阵。

$\tilde{M}_\alpha^i (\alpha \in \{1, 2, \dots, \psi\})$  是区间二型的第  $i$  个模糊集合,  $f_\alpha(x(k))$  是前件变量,  $i = 1, 2, \dots, p$ ,  $\psi$  是正标量;  $v(k)$  是一个标量信号且满足如下的统计特征:

$$E\{v(k)\} = 0, E\{v(u)v(w)\} = \begin{cases} \sigma_w^2, & u \neq w \\ 0, & u = w \end{cases}$$

其中,  $\sigma_w^2$  是常数。

这个随机变量  $\zeta(k)$  是满足伯努利分布的:

$$\text{Prob}\{\zeta(k) = 1\} = \bar{\zeta}, \text{Prob}\{\zeta(k) = 0\} = 1 - \bar{\zeta}$$

其中,  $\bar{\zeta} \in [0, 1]$  是一个已知标量, 实矩阵  $B(k)$  代表着参数不确定性且满足:

$$B(k) = MC(k)N \quad (1.2)$$

$M$  和  $N$  是适合维数的常数矩阵,  $C(k) \in R^{n_c \times n_c}$  是未知矩阵且满足:

$$C(k)^T C(k) \leq I$$

根据区间二型模糊建模准则, 上下隶属度函数定义为

$$\begin{cases} w_i^L(x(k)) = \prod_{p=1}^{\psi} \underline{\mu}_{\tilde{M}_p^i} f_p(x(k)) \\ w_i^U(x(k)) = \prod_{p=1}^{\psi} \bar{\mu}_{\tilde{M}_p^i} f_p(x(k)) \end{cases}$$

其中,  $w_i^U(x(k)) \geq w_i^L(x(k))$ 。

接下来, 引入状态相关的加权参数  $\underline{v}_i(x(k)) \geq 0$  和  $\bar{v}_i(x(k)) \geq 0$ 。

并且满足  $\underline{v}_i(x(k)) + \bar{v}_i(x(k)) = 1$ , 然后归一化的隶属度函数  $w_i(x(k))$  具有如下的形式:

$$w_i(x(k)) = \frac{w_i^L(x(k))\underline{v}_i(x(k)) + w_i^U(x(k))\bar{v}_i(x(k))}{\sum_{i=1}^p (w_i^L(x(k))\underline{v}_i(x(k)) + w_i^U(x(k))\bar{v}_i(x(k)))}$$

其中  $0 \leq w_i(x(k)) \leq 1$  和  $\sum_{i=1}^p w_i(x(k)) = 1$ 。

综上所述, 区间二型 T-S 模糊模型可以表示如下形式:

$$\begin{cases} x(k+1) = \sum_{i=1}^p w_i(x(k)) \left[ (A_i + \zeta(k)B(k))x(k) + F_i u(k) + D_i v(k) + H_i \sum_{h=1}^{\hbar} \mathcal{G}_h x(k-h) \right] \\ y(k) = \sum_{i=1}^p w_i(x(k)) C_i x(k) \\ z(k) = \sum_{i=1}^p w_i(x(k)) E_i x(k) \end{cases} \quad (1.3)$$

### 2.2. 网络攻击模型

本节将测量信号通过共享网络发送观测器, 其中信息传输可能被随机发生的 DOS 攻击恶意篡改, 表示如下

$$\mathfrak{R}(k) = \alpha(k)y(k) \quad (1.4)$$

其中,  $\mathfrak{R}(k) \in R^{n_y}$  表示当观测器受到攻击时的接收信号,  $\alpha(k)$  是一个伯努利随机变量, 利用下面的概率分布来调整随机发生的 DOS 攻击:

$$\text{Prob}\{\alpha(k) = 0\} = \bar{\alpha}, \text{Prob}\{\alpha(k) = 1\} = 1 - \bar{\alpha} \quad (1.5)$$

这里,  $\bar{\alpha} \in [0,1]$  且是一个已知常数。

### 2.3. 基于观测器的安全控制(OBSC)

为了减轻估计性质被随机发生的 DOS 攻击破坏, 股灾观测器中加入一个饱和函数, 具体来说, 抗异常值观测器的描述如下:

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^p w_i(x(k)) \left[ A_i \hat{x}(k) + F_i u(k) + H_i \sum_{h=1}^{\hbar} \mathcal{G}_h \hat{x}(k-h) + L_i \tau(\mathfrak{R}(k) - C_i \hat{x}(k)) \right] \\ \hat{z}(k) = \sum_{i=1}^p w_i(x(k)) E_i \hat{x}(k) \\ \hat{x}(j) = 0, j = \{-\hbar, \dots, -1, 0\} \end{cases} \quad (1.6)$$

其中, 这个饱和函数  $\tau(\cdot): R^{n_y} \rightarrow R^{n_y}$  被定义为:

$$\tau(l) = \left[ \tau(l_1) \ \tau(l_2) \ \dots \ \tau(l_{n_y}) \right]^T, \forall l \in R^{n_y} \quad (1.7)$$

$p = \{1, 2, \dots, n_y\}$ , 饱和函数  $\tau(\cdot)$  满足一个扇形有界条件:

$$\left( \tau(\beta_p) - \mu_p \beta_p \right)^T \left( \tau(\beta_p) - \mu_p \beta_p \right) \leq 0, (p = \{-\hbar, \dots, -1, 0\}) \quad (1.8)$$

其中,  $\beta_p$  是一个标量, 同时  $\mu_p$  也是一个标量且满足  $0 < \mu_p < 1$ 。

区间二型 T-S 模糊控制器的结构为:

控制器规则  $j$ : 如果  $g_1(x(k))$  是  $\tilde{N}_1^j$ , 和  $g_2(x(k))$  是  $\tilde{N}_2^j$ , 且  $\dots$ ,  $g_\psi(x(k))$  是  $\tilde{N}_\psi^j$ , 则有

$$u(k) = K_j \hat{x}(k) \quad (1.9)$$

其中, 控制器规则  $j$  是第  $j$  条模糊规则,  $g_\alpha(x(k))$ ,  $\alpha = \{1, 2, \dots, \psi\}$  是前件变量,  $K_j$  是控制器增益。

根据区间二型模糊建模准则, 上下隶属度函数定义为

$$\begin{cases} m_i^L(x(k)) = \prod_{p=1}^{\psi} \Theta_{\tilde{x}_p^i} \mathcal{G}_p(x(k)) \\ m_i^U(x(k)) = \prod_{p=1}^{\psi} \bar{\Theta}_{\tilde{x}_p^i} \mathcal{G}_p(x(k)) \end{cases}$$

其中,  $m_i^U(x(k)) \geq m_i^L(x(k))$ 。

接下来, 引入状态相关的加权参数  $\underline{v}_i(x(k)) \geq 0$  和  $\bar{v}_i(x(k)) \geq 0$ , 并且满足  $\underline{d}_i(x(k)) + \bar{d}_i(x(k)) = 1$ 。然后归一化的隶属度函数  $m_i(x(k))$  具有如下的形式:

$$m_i(x(k)) = \frac{m_i^L(x(k))\underline{d}_i(x(k)) + m_i^U(x(k))\bar{d}_i(x(k))}{\sum_{i=1}^p (m_i^L(x(k))\underline{d}_i(x(k)) + m_i^U(x(k))\bar{d}_i(x(k)))}$$

其中,  $0 \leq m_i(x(k)) \leq 1$  和  $\sum_{i=1}^p m_i(x(k)) = 1$ 。

综上所述, 区间二型 T-S 控制器可以表示为如下形式:

$$u(k) = \sum_{j=1}^p m_j(x(k)) K_j \hat{x}(k) \quad (1.10)$$

标记,  $\tilde{x}(k) = x(k) - \hat{x}(k)$ ,  $\tilde{z}(k) = z(k) - \hat{z}(k)$ , 这个估计误差可以得到:

$$\begin{cases} \tilde{x}(k+1) = \sum_{i=1}^p \sum_{j=1}^p w_i(x(k)) m_j(x(k)) \left[ A_i \tilde{x}(k) + D_i v(k) + \zeta(k) B(k) x(k) + H_i \sum_{h=1}^h \mathcal{G}_h \tilde{x}(k-h) - L_i \tau (\mathfrak{R}(k) - C_i \hat{x}(k)) \right] \\ \tilde{z}(k+1) = \sum_{i=1}^p w_i(x(k)) E_i \tilde{x}(k) \\ \tilde{x}(j) = \psi(j), j = \{-h, \dots, -1, 0\} \end{cases} \quad (1.11)$$

对  $\xi(k) = [x^T(k) \tilde{x}^T(k)]^T$ , 则有

$$\xi(k+1) = \sum_{i=1}^p \sum_{j=1}^p w_i(x(k)) m_j(x(k)) \left[ \mathcal{A}_{ij} + \bar{\zeta}(k) \mathcal{B}(k) \xi(k) + \mathcal{D}_i v(k) + \mathcal{H}_i \sum_{h=1}^h \mathcal{G}_h \xi(k-h) + \mathcal{L}_i \tau (\eta(k)) \right] \quad (1.12)$$

其中,

$$\mathcal{A}_{ij} = \begin{bmatrix} A_i + F_i K_j & -F_i K_j \\ 0 & A_i \end{bmatrix}, \quad \mathcal{B}(k) = \begin{bmatrix} B(k) & 0 \\ B(k) & 0 \end{bmatrix},$$

$$\mathcal{D}_i = \begin{bmatrix} D_i \\ D_i \end{bmatrix}, \quad \mathcal{H}_i = \begin{bmatrix} H_i & 0 \\ 0 & H_i \end{bmatrix},$$

$$\mathcal{L}_i = \begin{bmatrix} 0 \\ -L_i \end{bmatrix}, \quad C_{li} = [(\alpha(k)-1)C_i \quad C_i],$$

$$\bar{\zeta}(k) = \zeta(k) - \bar{\zeta}, \quad \bar{\alpha}(k) = \alpha(k) - \bar{\alpha},$$

$$\eta(k) = C_{li} \xi(k) = \mathfrak{R}(k) - C_i \hat{x}(k)$$

定义 1: 如果存在常数  $0 < \varepsilon < 1$ ,  $\alpha > 0$  和  $\bar{v} > 0$  以及系统动力学方程(1.12)式  $\xi(k)$  满足:

$$E\left\{\|\xi(k)\|^2\right\} \leq \varepsilon^k \alpha + v(k) \text{ 和 } \lim_{k \rightarrow \infty} v(k) = \bar{v}$$

则有上述系统(1.12)是均方意义上指数最终有界的。

定义 2: 如果系统是均方意义上指数最终有界的并且满足:

$$E\left\{\|\xi(k)\|^2\right\} \leq g, \quad \forall k \geq 0$$

则有上述系统(1.12)式是理想安全水平  $g$  的。

引理 1: 这个饱和函数在(1.8)  $\tau^\top(\eta(k))$  如果满足

$$\tau^\top(\eta(k))\tau(\eta(k)) + \eta^\top(k)U^\top\eta(k) - \eta^\top(k)(U^\top + I)\tau(\eta(k)) \leq 0$$

其中,  $U = \text{diag}\{\mu_1, \mu_2, \dots, \mu_q\}$

证明: 从(1.8)式不难看出

$$(\tau(p) - Up)^\top(\tau(p) - Up) = \sum_{q=1}^{n_q} (\tau(\alpha_q) - \mu_q \alpha_q) \leq 0$$

通过  $p = \eta(k)$ , 我们可以得到

$$\tau^\top(\eta(k))\tau(\eta(k)) + \eta^\top(k)U^\top\eta(k) - \eta^\top(k)(U^\top + I)\tau(\eta(k)) \leq 0$$

引理 2: 让  $Z = Z^\top, X, Y$  都是适合维数的实矩阵, 且  $H(k)$  满足  $H^\top(k)H(k) \leq I$ , 则

$$Z + XH(k)Y + Y^\top H^\top(k)X^\top < 0$$

当且仅当存在一个正标量  $\lambda > 0$ , 以至于

$$Z + \varepsilon XX^\top + \varepsilon^{-1}Y^{-1}Y < 0$$

或者

$$\begin{bmatrix} Z & \varepsilon X & Y^\top \\ \varepsilon X^\top & -\varepsilon I & 0 \\ Y & 0 & -\varepsilon I \end{bmatrix} < 0$$

引理 3: 存在  $N \in \mathbb{R}$  是正半定矩阵,  $X_m \in \mathbb{R}^n, Y_m > 0 (m = 1, 2, \dots)$ , 如果存在序列是收敛的, 则

$$\left(\sum_{m=1}^{\infty} Y_m X_m\right)^\top N \left(\sum_{m=1}^{\infty} Y_m X_m\right) \leq \left(\sum_{m=1}^{\infty} Y_m\right) \sum_{m=1}^{\infty} Y_m X_m^\top N X_m$$

### 3. 主要结果

定理 1:  $L_i$  是观测器增益,  $K_j$  是控制器增益, 如果存在正定矩阵  $P_1, Q_1$  和正标量  $\iota_1, \pi_1$  满足:

$$\Psi_2 < 0 \tag{1.13}$$

和

$$\frac{\bar{\theta} \lambda_{\max}(Q_1) h (\pi_1^h - 1) + (h+1) \bar{\lambda}}{\lambda_{\max}(P_1) (\pi_1 - 1)} \sup_{j \in \phi} E\left\{\|\varphi(j)\|^2\right\} + \frac{\pi_1}{\lambda_{\max}(P_1) (\pi_1 - 1)} \beta \leq g \tag{1.14}$$

其中,

$$\Psi_2 = \begin{bmatrix} \Psi_2^{11} & * & * \\ \Psi_2^{21} & \Psi_2^{22} & * \\ \Psi_2^{31} & \Psi_2^{32} & \Psi_2^{33} \end{bmatrix}$$

$$\begin{aligned}\Psi_2^{11} &= -P_1 + \bar{\theta}Q_1 + \mathcal{A}_h^T P_1 \mathcal{A}_h + \sqrt{\bar{\zeta}} \mathcal{A}_h^T P_1 \mathcal{B}(k) + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{A}_h + \bar{\zeta} \mathcal{B}^T(k) P_1 \mathcal{B}(k) - \iota_1 C_{li}^T U^T C_{li} \\ \Psi_2^{22} &= \mathcal{H}_i^T P_1 \mathcal{H}_i - \frac{1}{\theta} Q_1, \Psi_2^{33} = \mathcal{L}_i^T P_1 \mathcal{L}_i - \iota_1 I, \\ \Psi_2^{21} &= \mathcal{A}_h^T P_1 \mathcal{H}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{H}_i, \Psi_2^{31} = \mathcal{A}_h^T P_1 \mathcal{L}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{L}_i + \frac{1}{2} \iota_1 (U^T + I) C_{li}, \\ \Psi_2^{32} &= \mathcal{L}_i^T P_1 \mathcal{H}_i, \bar{\theta} = \sum_{h=1}^h \mathcal{G}_h, \bar{\lambda} = \max\{\lambda_{\max}(P_1), \lambda_{\max}(Q_1)\}, \beta = \lambda_{\max}(\mathcal{D}_i^T P_1 \mathcal{D}_i) \sigma_w^2,\end{aligned}$$

和对常数  $\pi_1 > 1$ , 满足

$$-\gamma\pi_1 + (\pi_1 - 1)\lambda_{\max}(P_1) + 2\bar{\theta}\lambda_{\max}(Q_1)\pi_1(\pi_1^h - 1) = 0$$

有  $\gamma = \lambda_{\min}(\Psi_2)$ 。

则有上述系统(1.12)是  $g$  安全水平。

证明: 首先构造李雅普诺夫函数

$$V(\xi(k)) = V_1(\xi(k)) + V_2(\xi(k)) \quad (1.15)$$

其中,

$$\begin{aligned}V_1(\xi(k)) &= \xi^T(k) P_1 \xi(k) \\ V_2(\xi(k)) &= \sum_{h=1}^h \mathcal{G}_h \sum_{r=k-h}^{k-1} \xi^T(r) Q_1 \xi(r) \\ \Delta V_1(\xi(k)) &= E\{V_1(\xi(k+1)) - V_1(\xi(k))\} \\ \Delta V_2(\xi(k)) &= E\{V_2(\xi(k+1)) - V_2(\xi(k))\}\end{aligned}$$

之后可得

$$\begin{aligned}E\{\Delta V_1(\xi(k))\} &= E\{V_1(\xi(k+1)) - V_1(\xi(k))\} \\ &= E\{\xi^T(k+1) P_1 \xi(k+1) - \xi^T(k) P_1 \xi(k)\} \\ &= E\left\{\left[\left(\mathcal{A}_h + \bar{\zeta} \mathcal{B}(k) + \bar{\zeta}(k) \mathcal{B}(k)\right) \xi(k) + \mathcal{D}_i v(k) + \mathcal{H}_i \sum_{h=1}^h \mathcal{G}_h \xi(k-h) + \mathcal{L}_i \tau(\eta(k))\right]^T P_1 \right. \\ &\quad \left. \times \left[\left(\mathcal{A}_h + \bar{\zeta} \mathcal{B}(k) + \bar{\zeta}(k) \mathcal{B}(k)\right) \xi(k) + \mathcal{D}_i v(k) + \mathcal{H}_i \sum_{h=1}^h \mathcal{G}_h \xi(k-h) + \mathcal{L}_i \tau(\eta(k))\right] - \xi^T(k) P_1 \xi(k)\right\} \\ &= E\left\{\xi^T(k) \left[\mathcal{A}_h^T P_1 \mathcal{A}_h - P_1 + (\bar{\zeta} + \bar{\zeta}(k)) \mathcal{A}_h^T P_1 \mathcal{B}(k) + (\bar{\zeta} + \bar{\zeta}(k)) \mathcal{B}^T(k) P_1 \mathcal{A}_h \right. \right. \\ &\quad \left. \left. + (\bar{\zeta} + \bar{\zeta}(k))^2 \mathcal{B}^T(k) P_1 \mathcal{B}(k)\right] \xi(k) + v^T(k) \mathcal{D}_i^T P_1 \mathcal{D}_i v(k) + \tau^T(\eta(k)) \mathcal{L}_i^T P_1 \mathcal{L}_i \tau(\eta(k)) \right. \\ &\quad \left. + \left(\sum_{h=1}^h \mathcal{G}_h \xi(k-h)\right)^T \mathcal{H}_i^T P_1 \mathcal{H}_i \left(\sum_{h=1}^h \mathcal{G}_h \xi(k-h)\right) + 2\xi^T(k) \left[\mathcal{A}_h^T P_1 \mathcal{H}_i + (\bar{\zeta} + \bar{\zeta}(k)) \mathcal{B}^T(k) P_1 \mathcal{H}_i\right] \right. \\ &\quad \left. \times \left(\sum_{h=1}^h \mathcal{G}_h \xi(k-h)\right) + 2\xi^T(k) \left[\mathcal{A}_h^T P_1 \mathcal{L}_i + (\bar{\zeta} + \bar{\zeta}(k)) \mathcal{B}^T(k) P_1 \mathcal{L}_i\right] \tau(\eta(k)) \right. \\ &\quad \left. + 2\left(\sum_{h=1}^h \mathcal{G}_h \xi(k-h)\right)^T \mathcal{H}_i P_1 \mathcal{L}_i \tau(\eta(k))\right\}\end{aligned}$$

$$\begin{aligned} &\leq \mathbb{E} \left\{ \xi^T(k) \left[ \mathcal{A}_\eta^T P_1 \mathcal{A}_\eta - P_1 + \sqrt{\bar{\zeta}} \mathcal{A}_\eta^T P_1 \mathcal{B}(k) + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{A}_\eta + \bar{\zeta} \mathcal{B}^T(k) P_1 \mathcal{B}(k) \right] \xi(k) \right. \\ &\quad + v^T(k) \mathcal{D}_i^T P_1 \mathcal{D}_i v(k) + \tau^T(\eta(k)) \mathcal{L}_i^T P_1 \mathcal{L}_i \tau(\eta(k)) + \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right)^T \mathcal{H}_i^T P_1 \mathcal{H}_i \\ &\quad \times \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right) + 2 \xi^T(k) \left[ \mathcal{A}_\eta^T P_1 \mathcal{H}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{H}_i \right] \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right) + 2 \xi^T(k) \\ &\quad \left. \times \left[ \mathcal{A}_\eta^T P_1 \mathcal{L}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{L}_i \right] \tau(\eta(k)) + 2 \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right)^T \mathcal{H}_i^T P_1 \mathcal{L}_i \tau(\eta(k)) \right\} \end{aligned}$$

此外, 在引理 3 的基础上, 我们可以得到如下:

$$\begin{aligned} \mathbb{E} \{ \Delta V_2(\xi(k)) \} &= \mathbb{E} \{ V_2(\xi(k+1)) - V_2(\xi(k)) \} \\ &= \mathbb{E} \left\{ \sum_{h=1}^h \mathcal{G}_h \sum_{r=k-h+1}^k \xi^T(r) Q_1 \xi(r) - \sum_{h=1}^h \mathcal{G}_h \sum_{r=k-h}^{k-1} \xi^T(r) Q_1 \xi(r) \right\} \\ &= \mathbb{E} \left\{ \bar{\theta} \xi^T(k) Q_1 \xi(k) - \sum_{h=1}^h \mathcal{G}_h \xi^T(k-h) Q_1 \xi(k-h) \right\} \\ &\leq \mathbb{E} \left\{ \bar{\theta} \xi^T(k) Q_1 \xi(k) - \frac{1}{\bar{\theta}} \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right)^T Q_1 \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right) \right\} \end{aligned}$$

从  $v(k)$  的统计特征, 可得,

$$\begin{aligned} \mathbb{E} \{ v^T(k) \mathcal{D}_i^T P_1 \mathcal{D}_i v(k) \} &\leq \lambda_{\max}(\mathcal{D}_i^T P_1 \mathcal{D}_i) \mathbb{E} \{ v^T(k) v(k) \} = \beta \\ \mathbb{E} \{ \Delta V(\xi(k)) \} &= \mathbb{E} \{ \Delta V_1(\xi(k)) \} + \mathbb{E} \{ \Delta V_2(\xi(k)) \} \\ &\leq \mathbb{E} \left\{ \xi^T(k) \left[ \mathcal{A}_\eta^T P_1 \mathcal{A}_\eta - P_1 + \bar{\theta} Q_1 + \sqrt{\bar{\zeta}} \mathcal{A}_\eta^T P_1 \mathcal{B}(k) + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{A}_\eta + \bar{\zeta} \mathcal{B}^T(k) P_1 \mathcal{B}(k) \right] \xi(k) \right. \\ &\quad + v^T(k) \mathcal{D}_i^T P_1 \mathcal{D}_i v(k) + \tau^T(\eta(k)) \mathcal{L}_i^T P_1 \mathcal{L}_i \tau(\eta(k)) + \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right)^T \left[ \mathcal{H}_i^T P_1 \mathcal{H}_i - \frac{1}{\bar{\theta}} Q_1 \right] \\ &\quad \times \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right) + 2 \xi^T(k) \left( \mathcal{A}_\eta^T P_1 \mathcal{H}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{H}_i \right) \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right) + 2 \xi^T(k) \\ &\quad \left. \times \left[ \mathcal{A}_\eta^T P_1 \mathcal{L}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{L}_i \right] \tau(\eta(k)) + 2 \left( \sum_{h=1}^h \mathcal{G}_h \xi(k-h) \right)^T \mathcal{H}_i^T P_1 \mathcal{L}_i \tau(\eta(k)) \right\} \\ &= \mathbb{E} \{ \mathfrak{S}_1^T(k) \Psi_1 \mathfrak{S}_1(k) \} + \beta \end{aligned}$$

其中,

$$\begin{aligned} \mathfrak{S}_1(k) &= \left[ \xi^T(k) \sum_{h=1}^h \mathcal{G}_h \xi^T(k-h) \tau^T(\eta(k)) \right]^T \\ \Psi_1 &= \begin{bmatrix} \Psi_1^{11} & * & * \\ \Psi_1^{21} & \Psi_1^{22} & * \\ \Psi_1^{31} & \Psi_1^{32} & \Psi_1^{33} \end{bmatrix} \\ \Psi_1^{11} &= -P_1 + \bar{\theta} Q_1 + \mathcal{A}_\eta^T P_1 \mathcal{A}_\eta + \sqrt{\bar{\zeta}} \mathcal{A}_\eta^T P_1 \mathcal{B}(k) + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{A}_\eta + \bar{\zeta} \mathcal{B}^T(k) P_1 \mathcal{B}(k), \\ \Psi_1^{22} &= \mathcal{H}_i^T P_1 \mathcal{H}_i - \frac{1}{\bar{\theta}} Q_1, \Psi_1^{33} = \mathcal{L}_i^T P_1 \mathcal{L}_i - \iota I, \end{aligned}$$



$$\Psi_1^{21} = \mathcal{A}_j^T P_1 \mathcal{H}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{H}_i, \Psi_1^{31} = \mathcal{A}_j^T P_1 \mathcal{L}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{L}_i, \Psi_1^{32} = \mathcal{L}_i^T P_1 \mathcal{H}_i,$$

然后, 在引理 1 的基础上, 可以得到

$$\begin{aligned} & \mathbb{E}\{\mathfrak{I}_1^T(k) \Psi_1 \mathfrak{I}_1(k)\} + \beta \\ & \leq \mathbb{E}\{\mathfrak{I}_1^T(k) \Psi_1 \mathfrak{I}_1(k)\} + \beta - \iota_1 \mathbb{E}\{\tau^T(\eta(k)) \tau(\eta(k))\} - \iota_1 \mathbb{E}\{\xi^T(k) C_{li}^T U^T C_{li} \xi(k)\} \\ & \quad + \iota_1 \mathbb{E}\{\xi^T(k) C_{li}^T (U^T + I) \tau(\eta(k))\} \\ & = \mathbb{E}\{\mathfrak{I}_2^T(k) \Psi_2 \mathfrak{I}_2(k)\} + \beta \end{aligned}$$

其中,

$$\begin{aligned} \mathfrak{I}_2(k) &= \left[ \xi^T(k) \sum_{h=1}^h \vartheta_h \xi^T(k-h) \tau^T(\eta(k)) \right]^T \\ \Psi_2 &= \begin{bmatrix} \Psi_2^{11} & * & * \\ \Psi_2^{21} & \Psi_2^{22} & * \\ \Psi_2^{31} & \Psi_2^{32} & \Psi_2^{33} \end{bmatrix} \\ \Psi_2^{11} &= -P_1 + \bar{\theta} Q_1 + \mathcal{A}_j^T P_1 \mathcal{A}_j + \sqrt{\bar{\zeta}} \mathcal{A}_j^T P_1 \mathcal{B}(k) + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{A}_j + \bar{\zeta} \mathcal{B}^T(k) P_1 \mathcal{B}(k) - \iota_1 C_{li}^T U^T C_{li} \\ \Psi_2^{22} &= \mathcal{H}_i^T P_1 \mathcal{H}_i - \frac{1}{\bar{\theta}} Q_1, \Psi_2^{33} = \mathcal{L}_i^T P_1 \mathcal{L}_i - \iota_1 I, \\ \Psi_2^{21} &= \mathcal{A}_j^T P_1 \mathcal{H}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{H}_i, \Psi_2^{31} = \mathcal{A}_j^T P_1 \mathcal{L}_i + \sqrt{\bar{\zeta}} \mathcal{B}^T(k) P_1 \mathcal{L}_i + \frac{1}{2} \iota_1 (U^T + I) C_{li}, \\ \Psi_2^{32} &= \mathcal{L}_i^T P_1 \mathcal{H}_i \end{aligned}$$

我们从(1.13)式可得

$$\mathbb{E}\{\Delta V(\xi(k))\} \leq -\gamma \mathbb{E}\{\|\mathfrak{I}_2(k)\|^2\} + \beta$$

接下来, 我们可以估计  $\mathbb{E}\{\|\xi(k)\|^2\}$  的上界, 从  $V(\xi(k))$  的概念中实现

$$V(\xi(k)) \leq \lambda_{\max}(P_1) \|\xi(k)\|^2 + \bar{\theta} \lambda_{\max}(Q_1) \sum_{r=k-h}^{k-1} \|\xi(r)\|^2$$

此外, 对于任何  $\rho > 1$ , 则有

$$\begin{aligned} & \mathbb{E}\{\rho^{k+1} V(\xi(k+1))\} - \mathbb{E}\{\rho^k V(\xi(k))\} \\ &= \rho^{k+1} \mathbb{E}\{\Delta V(\xi(k))\} + \rho^{k+1} \mathbb{E}\{V(\xi(k))\} - \rho^k \mathbb{E}\{V(\xi(k))\} \\ & \leq \rho^{k+1} \left( -\gamma \mathbb{E}\{\|\mathfrak{I}_2(k)\|^2\} + \beta \right) + \rho^k (\rho - 1) \left[ \lambda_{\max}(P_1) \mathbb{E}\{\|\xi(k)\|^2\} + \bar{\theta} \lambda_{\max}(Q_1) \sum_{r=k-h}^{k-1} \mathbb{E}\{\|\xi(r)\|^2\} \right] \\ & \leq \sigma_1(\rho) \rho^k \mathbb{E}\{\|\xi(k)\|^2\} + \sigma_2(\rho) \sum_{r=k-h}^{k-1} \mathbb{E}\{\|\xi(r)\|^2\} + \rho^{k+1} \beta \end{aligned}$$

其中,

$$\begin{aligned} \sigma_1(\rho) &= -\gamma \rho + (\rho - 1) \lambda_{\max}(P_1) \\ \sigma_2(\rho) &= (\rho - 1) \bar{\theta} \lambda_{\max}(Q_1) \end{aligned}$$

对于任何  $r > h$ , 两边求和从 0 到  $r-1$  项可得

$$\begin{aligned} & \mathbb{E}\{\rho^r V(\xi(r))\} - \mathbb{E}\{V(\xi(0))\} \\ & \leq \sigma_1(\rho) \sum_{k=0}^{r-1} \rho^k \mathbb{E}\{\|\xi(k)\|^2\} + \frac{\rho(1-\rho^r)}{1-\rho} \beta + \sigma_2(\rho) \sum_{k=0}^{r-1} \sum_{i=k-h}^{k-1} \rho^k \mathbb{E}\{\|\xi(i)\|^2\} \end{aligned}$$

而其中,

$$\begin{aligned} & \sum_{k=0}^{r-1} \sum_{i=k-h}^{k-1} \rho^k \mathbb{E}\{\|\xi(i)\|^2\} \\ & \leq \left( \sum_{i=h}^{-1} \sum_{k=0}^{i+h} 1 + \sum_{i=0}^{r-h-1} \sum_{k=i+1}^{i+h} 1 + \sum_{i=r-h}^{r-1} \sum_{k=i+1}^{r-1} 1 \right) \rho^k \mathbb{E}\{\|\xi(i)\|^2\} \\ & \leq \frac{\rho^h - 1}{\rho - 1} \sum_{i=-h}^{-1} \mathbb{E}\{\|\xi(i)\|^2\} + \frac{\rho(\rho^h - 1)}{\rho - 1} \sum_{i=0}^{r-1} \rho^i \mathbb{E}\{\|\xi(i)\|^2\} + \frac{\rho^h - 1}{\rho - 1} \sum_i \rho^i \mathbb{E}\{\|\xi(i)\|^2\} \end{aligned}$$

从而有

$$\begin{aligned} & \mathbb{E}\{\rho^r V(\xi(r))\} - \mathbb{E}\{V(\xi(0))\} \\ & \leq \sigma_1(\rho) \sum_{k=0}^{r-1} \rho^k \mathbb{E}\{\|\xi(k)\|^2\} + \frac{\rho(1-\rho^r)}{1-\rho} \beta + \sigma_2(\rho) \left[ \frac{\rho^h - 1}{\rho - 1} \sum_{i=-h}^{-1} \mathbb{E}\{\|\xi(i)\|^2\} \right] \\ & \quad + 2 \frac{\rho(\rho^h - 1)}{\rho - 1} \sum_{i=0}^{r-1} \rho^i \mathbb{E}\{\|\xi(i)\|^2\} \\ & \leq \sigma_3(\rho) \sum_{k=0}^{r-1} \rho^k \mathbb{E}\{\|\xi(k)\|^2\} + \frac{\rho(1-\rho^r)}{1-\rho} \beta + \sigma_4(\rho) \sup_{j \in \wp} \mathbb{E}\{\|\varphi(j)\|^2\} \end{aligned}$$

其中,

$$\begin{aligned} \sigma_3(\rho) &= \sigma_1(\rho) + \sigma_2(\rho) \frac{2\rho^{h+1} - 2\rho}{\rho - 1} \\ \sigma_4(\rho) &= \sigma_2(\rho) h \frac{\rho^h - 1}{\rho - 1} \end{aligned}$$

标记  $\sigma_3(\rho) = -\gamma < 0$  和  $\lim_{\rho \rightarrow \infty} \sigma_3(\rho) = +\infty$ , 可得存在一个标量  $\pi_1 > 1$ , 使得  $\sigma_3(\pi_1) = 0$ , 从而有

$$\mathbb{E}\{\pi_1^r V(\xi(r))\} - \mathbb{E}\{V(\xi(0))\} \leq \frac{\pi_1(1-\pi_1)}{1-\pi_1} \beta + \sigma_4(\rho) \sup_{j \in \wp} \mathbb{E}\{\|\varphi(j)\|^2\}$$

从定义项  $V(\xi(k))$  可知

$$\begin{aligned} \mathbb{E}\{V(\xi(0))\} & \leq (h+1) \bar{\lambda} \sup_{j \in \wp} \mathbb{E}\{\|\varphi(j)\|^2\} \\ \mathbb{E}\{\pi_1^r V(\xi(r))\} & \geq \lambda_{\min}(P_1) \pi_1^r \mathbb{E}\{\|\xi(r)\|^2\} \end{aligned}$$

从而有

$$\begin{aligned} \mathbb{E}\{\|\xi(r)\|^2\} & \leq \frac{1-\pi_1^r}{\lambda_{\min}(P_1) \pi_1^r (1-\pi_1)} \beta + \frac{\sigma_4(\pi_1) + (h+1) \bar{\lambda}}{\lambda_{\min}(P_1) \pi_1^r} \sup_{j \in \wp} \mathbb{E}\{\|\varphi(j)\|^2\} \\ & = \kappa^r \alpha \sup_{j \in \wp} \mathbb{E}\{\|\varphi(j)\|^2\} + T(\pi_1) \end{aligned}$$

其中,

$$\kappa = \frac{1}{\pi_1}, \quad \alpha = \frac{\sigma_4(\pi_1) + (\hbar + 1)\bar{\lambda}}{\lambda_{\min}(P_1)\pi_1^r}, \quad T(\pi_1) = \frac{1 - \pi_1^r}{\lambda_{\min}(P_1)\pi_1^r(1 - \pi_1)}\beta$$

根据定义 1, 最终上界为

$$\bar{T} = \lim_{r \rightarrow \infty} T(\pi_1) = \frac{\pi_1}{\lambda_{\min}(P_1)(\pi_1 - 1)}\beta$$

根据(1.14)从而有

$$E\left\{\|\xi(r)\|^2\right\} \leq \frac{\sigma_4(\rho) + (\hbar + 1)\bar{\lambda}}{\lambda_{\min}(P_1)\pi_1^r} \sup_{j \in \varphi} E\left\{\|\varphi(j)\|^2\right\} + \frac{\pi_1}{\lambda_{\min}(P_1)(\pi_1 - 1)}\beta \leq g$$

#### 4. 数值仿真

本文借助于一个数值仿真的例子来验证所提的方法有效性, 考虑到具有两条规则的区间二型模糊模型。

系统规则  $i$ : 如果  $f_1(x(k))$  是  $\tilde{M}_i^1$ , 则

$$\begin{cases} x(k+1) = (A_i + \zeta(k))x(k) + F_i u(k) + D_i v(k) + H_i \sum_{h=1}^{\hbar} \mathcal{G}_h x(k-h) \\ y(k) = C_i x(k) \\ z(k) = E_i x(k) \\ x(j) = \phi(j), \quad j = \{-\hbar, \dots, -1, 0\} \end{cases}$$

其中,  $\hbar = 3$

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.38 & 0.12 \\ 0.21 & 0.32 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0.13 & 0.28 \\ 0.06 & 0.51 \end{bmatrix} \\ C_1 &= [0.12 \quad 0.01], & C_2 &= [0.23 \quad 0.17] \\ D_1 &= \begin{bmatrix} 1.06 \\ 1.08 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.16 \\ 0.57 \end{bmatrix} \\ F_1 &= \begin{bmatrix} 0.08 & 0.02 \\ 0.06 & 0.04 \end{bmatrix}, & F_2 &= \begin{bmatrix} 0.28 & 0.12 \\ 0.15 & 0.14 \end{bmatrix} \\ E_1 &= \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0.12 \\ 0.32 \end{bmatrix} \\ H_1 &= \begin{bmatrix} 0.17 & 0.14 \\ 0.27 & 0.45 \end{bmatrix}, & H_2 &= \begin{bmatrix} 0.33 & 0.24 \\ 0.67 & 0.45 \end{bmatrix} \end{aligned}$$

隶属函数为:

其中一个为:

$$\begin{cases} w_1^L(x_1) = 1 - e^{x_1^2/0.78} \\ w_1^U(x_1) = 1 - 0.22e^{x_1^2/0.54} \\ w_1^L(x_1) = 1 - w_1^U(x_1) \\ w_1^U = 1 - w_1^L(x_1) \end{cases}$$

另一个为:

$$\begin{cases} m_1^L(x_1) = 1 - \frac{1}{1 + e^{x_1+5-1}} \\ m_1^U(x_1) = 1 - \frac{1}{1 + e^{x_1+5+1}} \\ m_1^L(x_1) = 1 - m_1^U(x_1) \\ m_1^U(x_1) = 1 - m_1^L(x_1) \end{cases}$$

根据以上的数据画出相应的效果图:

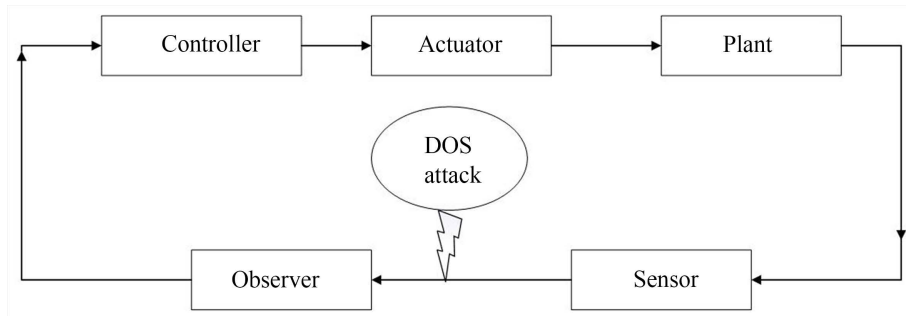


Figure 1. System structure diagram  
图 1. 系统结构图

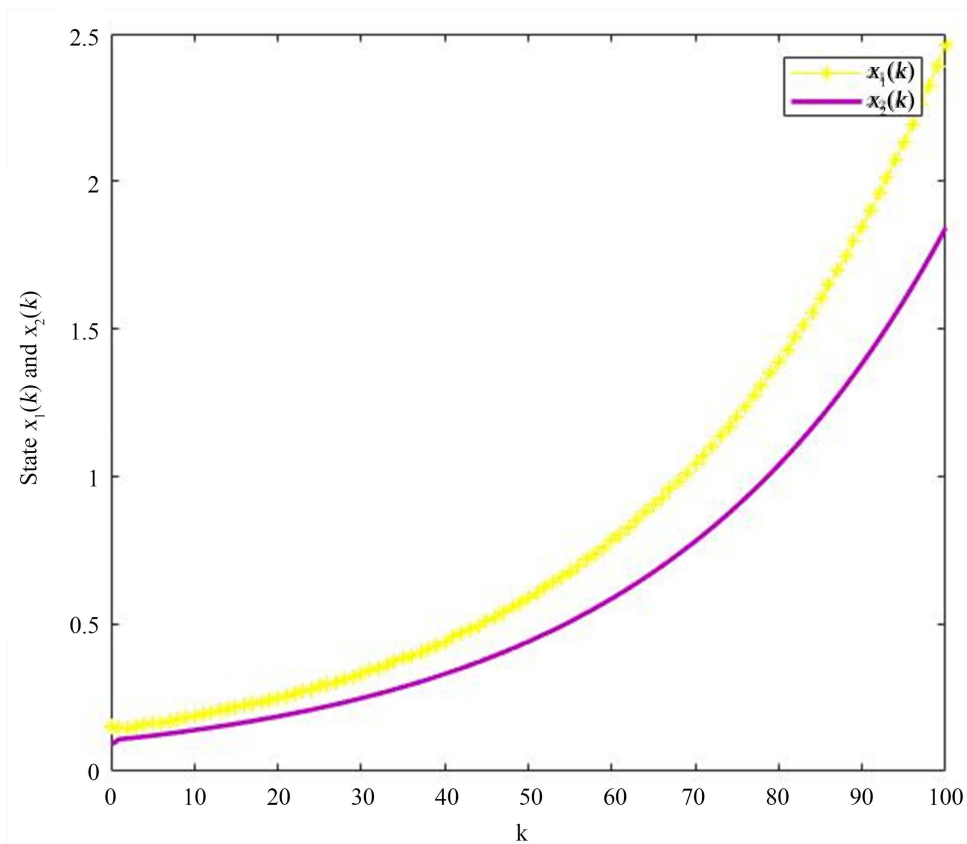
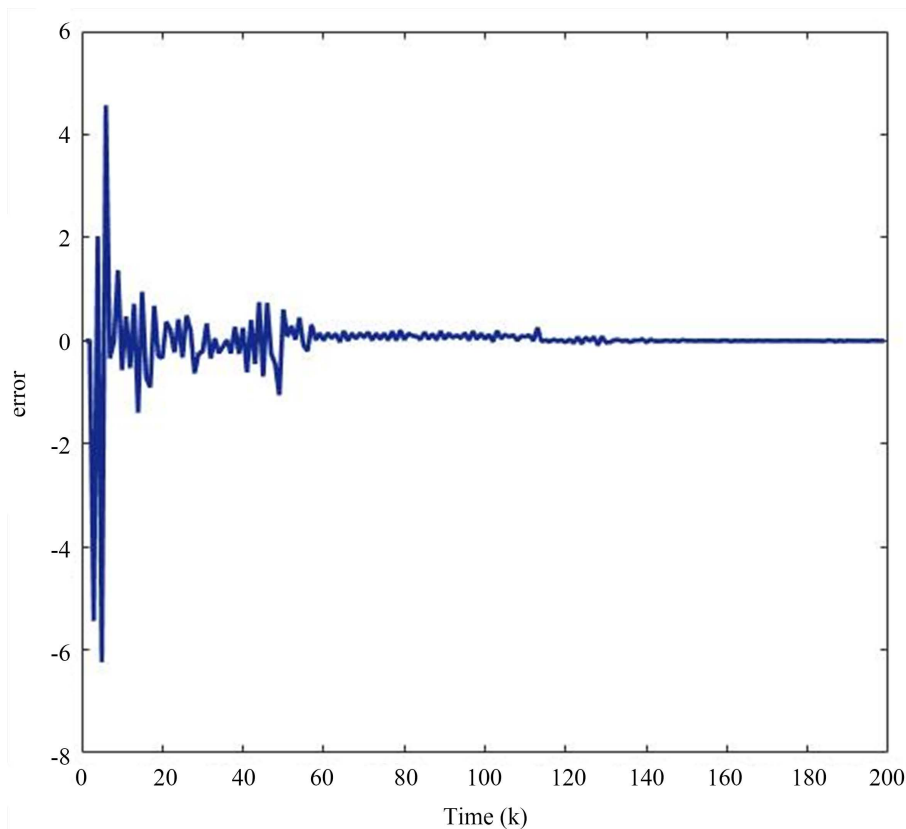


Figure 2. Evolution of the system  $x(k)$  without control  
图 2. 开环控制下系统状态  $x(k)$



**Figure 3.** Evolution of the system  $x(k)$  with control

**图 3.** 闭环控制下系统状态  $x(k)$

图 1 是系统结构图, 有一个未知的非线性装置、一个传感器、一个观测器、一个控制器和一个执行器组成, 本文就是图 1 的基础上进行研究的。图 2 是开环状态下系统的状态图, 随着时间的增加, 系统的状态  $x(k)$  趋于无穷大, 是属于发散的且不稳定。图 3 是闭环控制下的系统状态, 随着时间的增加, 系统的误差逐渐趋于零, 这说明估计的性能比较好。故在闭环控制下系统的性能比较稳定, 开环下是发散的, 不稳定。

## 5. 总结

本文研究了 DOS 攻击下具有分布时滞的 IT2 T-S 模糊离散时间系统的抗离群观测器控制问题。在李雅普诺夫稳定性理论的支持下, 导出了保证均方意义上估计误差有界性和安全级别的充分条件。利用正交分解和线性矩阵不等式技术, 将控制问题简化为线性凸优化问题。数值仿真结果表明了所提控制方法的有效性和可靠性。需要指出的是, 本文的主要成果可以推广到未来的其他研究课题, 如更复杂的网络诱导现象或饱和约束的发生。

## 参考文献

- [1] Yang, X.Z., Wu, L.G. and Lam, H.K. (2014) Stability and Stabilization of Discrete-Time T-S Fuzzy System with Stochastic Perturbation and Time-Varying Delay. *IEEE Transactions on Fuzzy System*, **22**, 124-138. <https://doi.org/10.1109/TFUZZ.2013.2249518>
- [2] Zhao, T., Liu, J.H. and Dian, S.Y. (2019) Finite-Time Control for Interval Type-2 Fuzzy Time-Delay Systems with Norm-Bounded Uncertainties and Limited Communication Capacity. *Information Sciences*, **483**, 153-173.

- <https://doi.org/10.1109/TFUZZ.2013.2249518>
- [3] Wit, E. and McClure, J. (2004) Statistics for Microarrays: Design, Analysis, and Inference. 5th Edition, John Wiley & Sons Ltd., Chichester, 5-18.
- [4] Kau, S.W., Lee, H.J. and Yang, C.M. (2007) Robust  $H_\infty$  Fuzzy Static Output Feedback Control of T-S Fuzzy Systems IEEE Transaction on Systems with Parametric Uncertainties. *Fuzzy Sets and Systems*, **158**, 135-146. <https://doi.org/10.1016/j.fss.2006.09.010>
- [5] Lam, H.K. and Seneviratne, L.D. (2008) Stability Analysis of Interval Type-2 Fuzzy-Model-Based Control Systems. *IEEE Transaction on Systems, Man, and Cybernetics*, **38**, 617-628. <https://doi.org/10.1109/TSMCB.2008.915530>
- [6] Zhang, Z.N., Niu, Y.G. and Song, J. (2020) Input-to-State Stabilization of Interval Type-2 Fuzzy Systems Subject to Cyber Attacks: An Observer-Based Adaptive Sliding Mode Approach. *IEEE Transactions on Fuzzy Systems*, **28**, 190-203. <https://doi.org/10.1109/TSMCB.2008.915530>
- [7] Pan, Y.N. and Yang, G.H. (2021) Event-Driven Fault Detection for Discrete-Time Interval Type-2 Fuzzy Systems. *IEEE Transaction on Systems, Man, and Cybernetic: Systems*, **51**, 4959-4968. <https://doi.org/10.1109/TSMCB.2008.915530>
- [8] Li, H.Y., Wu, L., Lam, H.-K. and Gao, Y. (2015) Filtering of Interval Type-2 Fuzzy Systems with Intermittent Measurements. *IEEE Transaction on Cybernetics*, **46**, 668-678. <https://doi.org/10.1109/TCYB.2015.2413134>
- [9] Zhang, M., Shi, P., Shen, C. and Wu, Z.-G. (2019) Static Output Feedback Control of Switched Nonlinear Systems with Actuator Faults. *IEEE Transaction on Fuzzy Systems*, **28**, 1600-1609. <https://doi.org/10.1109/TFUZZ.2019.2917177>
- [10] Wu, H.N. (2018) An ILMI Approach to Robust  $H_2$  Static Output Feedback Fuzzy Control for Uncertain Discrete-Time Nonlinear System. *Automatic*, **44**, 2333-2339. <https://doi.org/10.1016/j.automatica.2008.01.002>